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Spatial Compressive Sensing for Strain Data Reconstruction from Sparse Sensors

by Mulugeta A Haile

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14. ABSTRACT

Compressed sensing, also known as compressive sensing, is a technique for finding sparse solutions to underdetermined linear systems. In signal processing, compressed sensing is the process of acquiring and reconstructing a signal that is known to be sparse or compressible. The theory asserts that certain signals or images can be recovered from what was previously thought to be highly incomplete measurements. This report reviews this rapidly growing field and its application in structural health monitoring. The report also presents a theoretical problem of constructing the full-field strain similar to that traditionally obtained from digital image correlation using very few samples of discrete strain measurements.

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sparsity, measurement matrix, suboptimal, L1-norm, full-field strain, restricted isometry property

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1. Introduction

There is a need to develop new sensing technology to acquire structural diagnostic information more efficiently using a suboptimal number of sensors. The desire to limit the number of sensors is in part due to the following issues[1]:

- *High deployment cost*. The cost of installing a large number of sensors and the attendant cabling has always been a major cost driver of structural health monitoring (SHM) systems.

- *Altered structural or mechanical integrity*. Emplacing or embedding a large number of sensors may reduce the capacity of load-bearing structures.[2]

- *Lack of feasible sensor placement region*. Complex geometry components may not have feasible regions where sensors can be emplaced or embedded.

There are also, however, valid reasons to use a large number of sensors:

- *Large area of a structure is monitored.* The number of distributed sensors increases with the spatial size of the monitored area or the area to be covered.

- *Improved detection accuracy.* Accurate damage detection may require fusion of information collected from several heterogeneous sensors.

Combining these 2 conflicting needs reveals that the existing SHM sensing technology cannot meet the requirements of low-cost and zero-miss damage detection systems.

Consider, for example, the problem of collecting full-field strain information from a composite shaft, as shown in Fig. 1. The figure shows a composite tube with 5 strain sensors on the left-hand side; these sensors are used to monitor the condition of the tube in a conventional SHM paradigm. On the right-hand side is full-field strain image obtained using a digital image correlation nondistractive inspection technique. A damage pattern recognition system built with the SHM system of the tube using the full-field information will be more robust than one using data from the 5 strain sensors alone. Full-field measurements enable detection of heterogeneities in a strain, stress, or temperature field that can't be seen using conventional sensor measurements. However, acquiring a full-field strain image using strain sensors requires strain gages to be placed at every pixel point on the tube, which is not feasible.
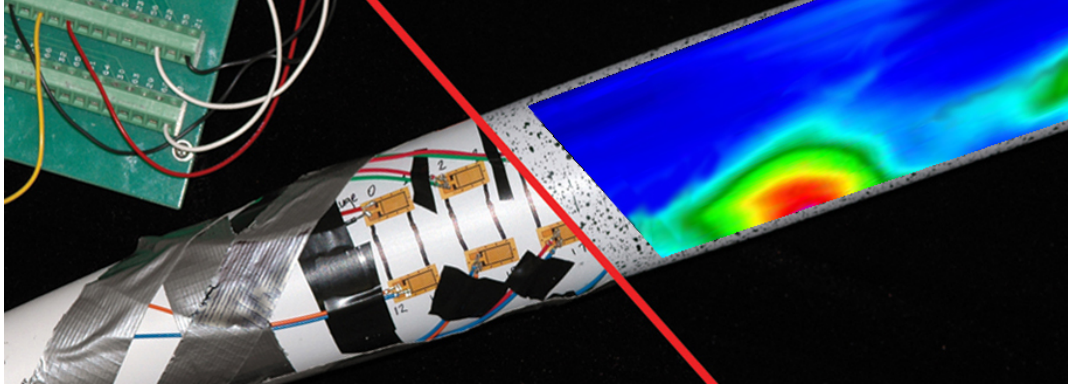
Fig. 1   Strain sensors and full-field digital image correlation

The objective of this research is to develop fundamentally new sensing technology using the novel theory of compressive sensing and principles of continuum mechanics. Compressive sensing, also known as compressed sensing, refers to the theory that, for certain types of signals, a small number of nonadaptive samples carries sufficient information to approximate the signal. The theory asserts that certain signals or images can be recovered from what was previously believed to be a highly incomplete measurement. Compressed sensing is a relatively new theory with a potential to address problems in sensor technology and similar areas of applications. The link between compressed sensing and SHM can be established as follows. It is well known that structural damage is a sparse event, and the mechanical metrics, such as stress, strain, etc., whose perturbation presages damage is a superposition of a finite number of spikes resulting from discontinuities (such as cracks) in the structure. The aim of SHM is to quantify and locate the spikes in the structure. Compressed sensing, at least theoretical, can provide information about this spiky data using a limited number of sensors that directly acquire compressed data. This work is the first attempt to implement compressed sensing in SHM. If implemented successfully, the method would enable diagnosis over a wide inspection area using far fewer sensors than traditionally possible.

Sections 1–4 of this report deal with the theoretical aspects of compressive sensing. A detailed description of the mathematical formulation and practical definition of signals that can be recovered using compressive sensing are provided. Sections 5–8 outline the implementation of the compressive sensing technique in structural health monitoring. A numerical experiment showing reconstruction of full-field strain data from discrete strain measurements is presented. The example assumes that a critical load-carrying member is instrumented with a network consisting of suboptimal strain-measuring sensors (e.g., fiber optic sensors with Bragg gratings) and that strain data is only available at far fewer discrete locations than would be required to construct full-field information.

2. Theory

Compressive sensing relies on the empirical observation that certain types of signals or images can be well-approximated by a sparse expansion in terms of a suitable basis or frame.[3,4] This means that the expansion has only a small number of significant terms or, in other words, that the coefficient vector can be well-approximated with one having only a small number of nonvanishing entries.[5,6] Graphically, as shown in Fig. 2, the N-sampled data can be approximated by only M-samples (or measurements), such that M << N provided the data is k-sparse.[7]
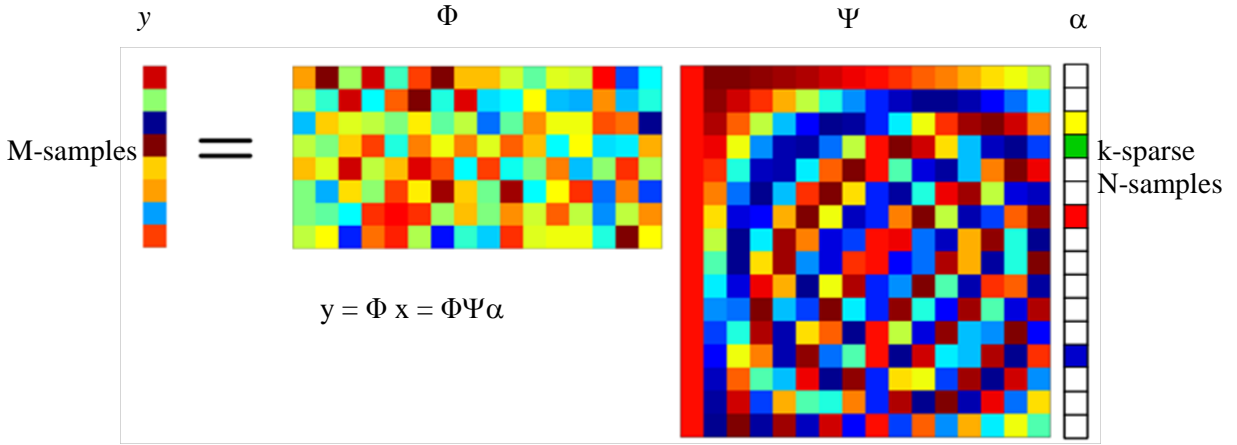

Fig. 2   Compressive sensing

Mathematically, for a given basis function $\{\psi_i\}_{i=1}^{N}$ a signal $x \in R^N$ can be represented by N coefficients $\{\alpha_i\}_{i=1}^{N}$ as $x = \sum_{i=1}^{N} \alpha_i \psi_i$ or in a vector form $x = \Psi\alpha$. A signal x is *k-sparse* if only $K << N$ entries of x are nonzero. The set of indices corresponding to the nonzero entries are called the support of x and are denoted by $supp(x) = \{j : x_j \neq 0\}$ and

$$\|\mathbf{x}\|_{\mathbf{0}} = |supp(x)|. \tag{1}$$

|supp(x)| denotes the cardinality of supp(x). For a *k*-sparse signal x, the l_0-quasinorm $\|x\|_0 \leq k$ and for $k \in \{1, 2, \ldots, N\}$, the set of all *k*-sparse signals is the union of the $\binom{N}{K}$ k-dimensional subspaces aligned with the coordinate axes in R^N. Such a union of subspaces can be denoted by Σ_k

$$\Sigma_k := \left\{ x \in R^N : \|x\|_0 \leq k \right\}. \tag{2}$$

Furthermore, the best *k-term* expansion of the signal $x \in \mathrm{R}^N$ in l_p (Lp-norm) is defines as

$$\sigma_k(x)_p = \inf_{x_k \in \Sigma_k} \|x - x_k\|_p . \tag{3}$$

x is said to be compressible if $\sigma_k(x)$ decays quickly in *k*. Indeed, in order to compress *x* one may simply store only the *k* largest coefficients. When *x* is reconstructed from its compressed version, the nonstored entries are simply set to zero, and the reconstruction error $\sigma_k(x)_p$ is often very small.

Many natural and manmade signals are not strictly sparse but can be approximated as such and are, therefore, compressible. Consider a signal *x* whose coefficients, when sorted in order of decreasing magnitude, are $r(x) = \left(|x_{i_1}|, \ldots, |x_{i_N}|\right)^T$, such that $|x_{i_j}| \geq |x_{i_{j+1}}|$ for j = 1, . . . , N − 1 decay according to the power law[8]

$$\left|x_{r(i)}\right| \leq Si^{-\frac{1}{r}}, \quad i = 1, \ldots, N . \tag{4}$$

Owing to the rapid decay of the sorted coefficients, such signals are well approximated by *k*-sparse signals. Let $x_k \in \Sigma_k$ represent the best *k*-term approximation of *x*, which is obtained by keeping just the first *k* terms in $x_{r(i)}$. Then the reconstruction error in the l_p sense is

$$E_k(x)_p := \arg\min_{\bar{x} \in \sigma_K} \|x - \bar{x}\|_p = \|x - x_K\|_p , \tag{5}$$

where the l_p norm of the vector *x* is defined as

$$\|x\|_n = \left(\sum_{i=1}^{N} |x_i|^p\right)^{\frac{1}{p}} . \tag{6}$$

Then the error[7]

$$E_k(x)_p \leq (rs)^{-\frac{1}{p}} Sk^{-s} \tag{7}$$

with $s = \frac{1}{r} - \frac{1}{p}$. That is, when measured in the l_p norm, the signal's best approximation error has a power-law decay with exponent *s* as *k* increases. Such a signal is denoted as *s-compressible.* In the traditional signal compression framework, as shown in Fig. 3, one generally acquires the full *N*-sample signal *x*; compute the complete set of transform coefficients $\alpha = \Psi^{-1}x$; locate the *k* largest terms and discard the (N − k) smallest coefficients; and encode the *k* values and locations of the largest coefficients.
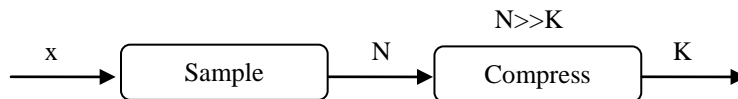

Fig. 3   Traditional sample-then-compress data acquisition

While widely used in transform coding,[9,10] this sample-then-compress strategy has several drawbacks. First, much effort is spent obtaining full information on the signal. Second, the encoder must compute all of the N transform coefficients α, even though it will discard all but k of them. What would be novel is to be able to obtain the compressed version of the signal directly (Fig. 4) by taking only a small number of measurements or samples. It is not obvious at all whether this is possible since measuring directly the large coefficients requires a priori knowledge of the support of the largest transform coefficients. Interestingly, for a certain type of signals, compressive sensing provides a technique of reconstructing a compressed version of the original signal by taking only a small amount of linear and nonadaptive measurements. The exact number of required compressed samples is comparable to the compressed size of the original signal.
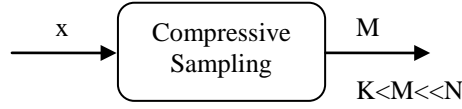
x → Compressive Sampling → M

K<M<<N

Fig. 4   Directly acquire compressed data

3.   Matrix Completion

An interesting class problem that is similar to compressive sensing is matrix completion. In matrix completion, one would like to recover a data matrix M with n_1 rows and n_2 columns by observing only m of its entries, which is comparably much smaller than the total number of entries $n_1 n_2$. A lot of real-world models can be categorized as matrix completion problems, provided the matrix describing the model is known to be structured in the sense that it is low rank. The general form of the problem is[11]

$$\min_{X} rank(X) \text{ subject to } X_{ij} = M_{ij} \text{ for } (i, j) \in \Omega, \tag{8}$$

where X is the decision variable, and rank (X) is equal to the rank of the matrix X. Eq. 7, unfortunately, has little practical use because this optimization problem is NP-hard (nondeterministic polynomial time hard), and known algorithms require time doubly exponential in the dimension n of the matrix. Hence, a heuristic alternative that minimizes the sum of the singular values over the constraint set is considered. This sum is called the nuclear norm and is given by[11]

$$\|X\|_* = \sum_{k=1}^{n} \sigma_k(X), \tag{9}$$

where $\sigma_k(X)$ denotes the kth largest singular value of X. The heuristic optimization of Eq. 8 is then given by[11]

$$\min \|X\|_* \quad \text{subject to} \quad X_{ij} = M_{ij}. \tag{10}$$

The rank function counts the number of nonvanishing singular values, whereas the nuclear norm sums their amplitude and, in some sense, is to the rank functional to what the convex l_1 norm is to the counting l_0 norm in compressive sensing. The matrix completion problem is quite similar to compressive sensing, as a similar heuristic approach, convex relaxation, is used to recover structured data. Matrix completion solves the nuclear norm of X to solve the rank minimization problem, while compressive sensing uses the l_1-norm as a relaxation of the l_0 counting problem.

One practical application of matrix completion is in the triangulation of sensor data from incomplete measurements. Here, partial information is provided about the distances between sensors and reconstruction of the low-dimensional geometry describing their locations. For example, we may have a network of sensors scattered randomly across a region of a structure as shown in Fig. 5. Suppose each sensor only has the ability to construct distance estimates from its nearest fellow sensors. From these noisy distance estimates, we can form a partially observed distance matrix. We can then estimate the true distance matrix whose rank will be equal to 2 if the sensors are located in a plane or 3 if they are located in 3-dimensional (3-D) space. In this case, we need to observe only a few distances per node to have enough information to reconstruct the positions of the objects.
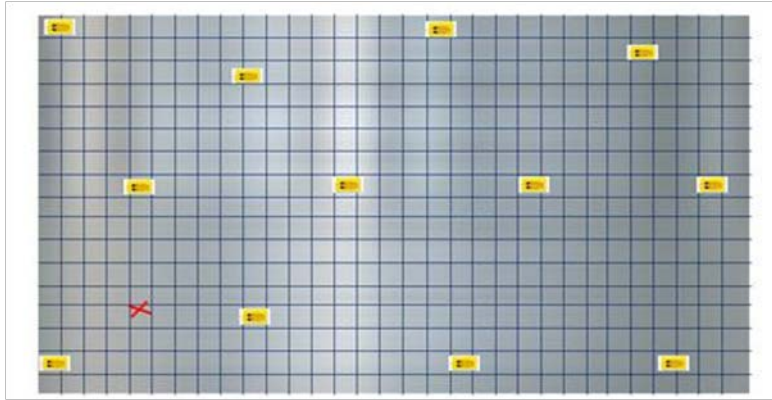

Fig. 5   Spatially distributed strain sensors on a gridded plate

4.   Compressed Samples

Compressive sensing integrates the signal acquisition and compression steps into a single process. In compressive sensing, one does not acquire x directly but rather acquires $M < N$ linear measurements of $y = \Phi x$ using an $M \times N$ measurement matrix Φ (also known as compressive sensing matrix). Normally, it is an underdetermined ill-posed problem to recover x from partial measurements y. To make it possible, compressive sensing relies on the assumptions of sparsity and incoherence.

Sparsity: Let $x \in \mathrm{R}^N$ be an unknown vector (depending on context, a digital image or signal) and Ψ a fixed dictionary such as Fourier, wavelets, or curvelets, then x can be decomposed into $x = \Psi\alpha$, where α is the vector of coefficients that represents x on Ψ. The unknown vector x is said to be sparse if all but a few entries of α are zero or they can be discarded without much loss of information.[4,7]

Incoherence: For a given measurement matrix Φ, compressive sampling requires that Φ and the sparsity basis Ψ be as incoherent (dissimilar) as possible. A measurement of coherence between Φ and Ψ is given by[4,7]

$$\mu(\Phi, \Psi) = \sqrt{N} \max_{1 \le i \le n, 1 \le j \le N} \left| \langle \varphi_i, \psi_j \rangle \right|, \tag{11}$$

where i, j denotes the index of columns in each matrix. Roughly, Eq. 10 implies that the basis $\{\psi_j\}$ cannot sparsely represent the vector $\{\varphi_i\}$ with the parameter $\mu \in \left[1, \sqrt{N}\right]$ measuring the maximal correlation between the 2 matrices. In general, random sampling matrixes are incoherent with most known fixed transform bases. The incoherence between the sampling matrices and sparse transform indicates that one can get new information from sampling, which is not already represented by the known dictionary Ψ since the measurements are global. Consider that $y = \Phi x = \Phi\Psi\alpha$ be the vector of n measurements of the sparse signal x, with the number of nonzero coefficients,

$$S = \|\alpha\|_0 \ll m \ll N, \tag{12}$$

where there are many more unknowns than measurements. Then it turns out that if the number of measurements m satisfies,

$$m \ge C \cdot \mu^2(\Phi, \Psi) \cdot S \cdot \log(N). \tag{13}$$

then, the original signal x can be reconstructed exactly from y with overwhelming probability by solving a convex problem,[4,5,6]

$$\hat{\alpha} = \arg\min\|\alpha\|_1 \quad \text{subject to } y = \Phi\Psi\alpha, \tag{14}$$

where $\|x\|_1 = \sum_{i=1}^{N}|x_i|$. By solving Eq. 13, one seeks the sparsest coefficients among all possible α, satisfying $y = \Phi\Psi\alpha$. If the solution coincides with α, one can get a perfect reconstruction of the original unknown signal.

4.1 Restricted Isometry Property

To have a unique solution $\hat{\alpha}$ to the l_1 minimization problem Eq. 13, the matrix $\Phi\Psi$ holds the restricted isometry property (RIP).

Definition 1. The restricted isometry constant δ_k of a matrix $\Phi\Psi \in \mathrm{R}^{m \times N}$ is the smallest number such that

$$\left(1-\delta_k\right)\|x\|_2^2 \leq \|\Phi x\|_2^2 \leq \left(1+\delta_k\right)\|x\|_2^2 \quad \text{for all} \ \ x \in \Sigma_k. \tag{15}$$

A matrix $\Phi\Psi$ is said to satisfy the RIP of order k with constant δ_k if $0 \leq \delta_k \leq 1$. The k-RIP ensures that all submatrices of Φ of size $M \times k$ are close to an isometry, and therefore distance and information are preserved. In general, most realistic recovery algorithms typically require that $\Phi\Psi$ have a slightly higher-order RIP (such as $2k$-RIP, $3k$-RIP) in order to preserve distances between k-sparse vectors and other higher-order structures. While the design of a measurement matrix $\Phi\Psi$ satisfying the k-RIP is an NP-complete problem,[12] random matrices whose entries are i.i.d. Gaussian, Bernoulli, and random Fourier work with high probability provided the number of compressed measurements $M = O(k \log(N/k))$.

Remark 1. A Gaussian matrix A has entries chosen as i.i.d. random variables with expectation 0 and variance 1/m. The Gaussian matrix has an optimal RIP with

$$\delta_r \leq 0.1 \ \ \Rightarrow \ \ m \geq Cr \log\left(N/r\right). \tag{16}$$

Remark 2. A Bernoulli matrix A has entries of $\pm 1/\sqrt{m}$ Bernoulli random variables with equal probability.

Gaussian and Bernoulli matrices provide optimal conditions for sparse recovery of compressible signals using partial measurements. However, they are of somewhat limited use for practical applications where one rarely has the freedom to inject randomness into the measurements. Most practical applications impose physical or other constraints rather than randomness. A very important class of a structured random compressive sensing matrix is the partial Fourier matrix.

Remark 3. A random partial Fourier matrix $\Phi \in C^{m \times N}$ is derived from the discrete Fourier matrix $F \in C^{N \times N}$ with entries

$$F_{ij} = \frac{1}{\sqrt{N}} e^{2\pi jk/N} \tag{17}$$

by selecting m rows uniformly at random among all N rows.

Theorem 1. Let $\Phi \in C^{m \times N}$ be the random partial Fourier matrix as just described. Then the restricted isometry constant of the rescaled matrix $\sqrt{N/m}A$ satisfies $\delta_k \leq \delta$ with a probability of at least $1 - N^{-\gamma \log^3(N)}$, provided

$$m \geq C\delta^{-2}k \log^4\left(N\right), \tag{18}$$

where C and $\gamma > 1$ are constants. Combining this estimate with the l_1-minimization results shows

that recovery with high probability can be ensured for all *k*-sparse *x*, provided

$$m \geq Ck \log^4(N). \tag{19}$$

The proof of this theorem is available in Candes et al.[4] and Tropp and Gilbert.[13] Even though partial Fourier matrices may require additional samples to achieve a small restricted isometry constant, they are more interesting for the following reasons:[8]

- There are technologies that acquire random Fourier measurements at unit cost per sample, such as MRI.

- The sampling matrix can be applied to a vector in time $O(N \log N)$.

- The sampling matrix requires only $O(m \log N)$ storage.

These traits are essential for the translation of compressive sampling from theory into practice.

5. Application in Structural Health Monitoring

The integrity of airframe structures gradually degrades because of slow-growing damage, such as crack propagation and corrosion. In general, such damage propagates monotonically, and the degradation increases the risk of operation and the cost of maintenance. To reduce cost, avert sudden component failure, and increase readiness, the degradation must be monitored. Structural health monitoring (SHM) is the process of detecting and locating damage in structures. The main goal of SHM is to improve the safety and reliability of structures by providing damage information before it reaches a critical state. To achieve this goal, various sensing technologies are being developed.

The data acquisition portion of SHM involves selecting sensor type, sensor number, sensor location, and the transmitting hardware. In general, sensors do not "see" damage directly; instead, they sense the presence of damage indirectly by measuring adverse changes in structural response, such as loss of stiffness or excessive deformation and strain. Real-time reconstruction of the full-field deformation or strain is essential to provide a global state awareness of airframe structures. To perform strain-based SHM,[14] the load-carrying component will be instrumented with a network of strain sensors, such as optical fiber sensors with Bragg gratings or strain gages. The full-field reconstruction of the strain field at every material point of the structure from a set of discrete point strain measurements represents an interesting problem that has never been solved before.

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6. Full-Field Strain

As discussed in the first half of this report, compressive sensing relies on the empirical observation that certain types of signals or images can be well-approximated by a sparse expansion in terms of a suitable basis or frame. This means that the expansion has only a small number of significant terms or, in other words, that the coefficient vector can be well-approximated with one having only a small number of nonvanishing entries. This observation has a direct implication in the number of measurement nodes (sensors) needed in SHM. A typically SHM observes a system over time using periodically sampled dynamic measurements or mechanical metrics (such as stress, deformation, temperature, acceleration, etc.). The structure of these metrics is governed by certain physical and empirical laws, such as continuum mechanics, second law of thermodynamics, and constitutive theories. If damage, such as a crack, occurs in a structure, the damage metric, such as the local strain field, shows spikes at or near the location of the damage (usually at the crack tip [also known as crack-tip singularity]), but farther away, the spiky metrics decay quickly, resulting in a smooth field, as shown in Fig. 6.
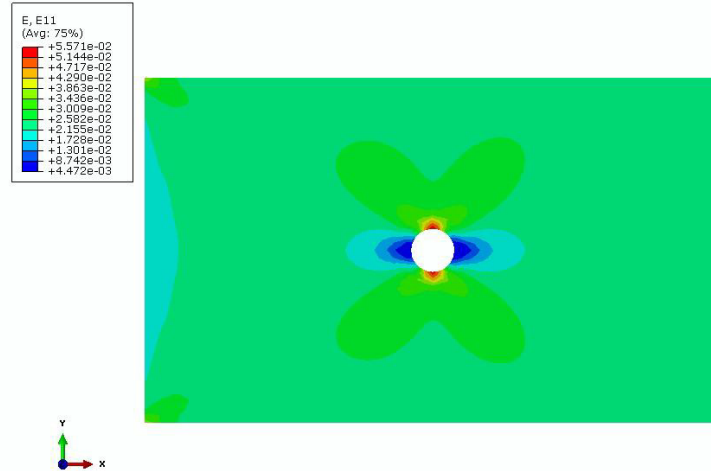

Fig. 6   Finite element model showing the principal strain field on a rectangular plate

The strain image shown in Figs. 6 or 7 is spiky. Such a spiky image often has a sparse representation in certain orthonormal bases. Figure 8 shows the frequency domain (Fourier basis) representation of the same strain image shown in Fig. 6. As can be seen on the fast-Fourier transform image, only a few large Fourier coefficients are needed to represent the strain field, and, as such, if one can directly acquire these large Fourier coefficients, it is possible that one can recover the entire strain image from very few measurements. Traditionally, however, if one needs to obtain the full-field strain, unless it is digital image correlation as in Fig. 1, one has to emplace sensors (strain gage) at every pixel point of the plate, which requires strain gages on the order of 10^3 or more.
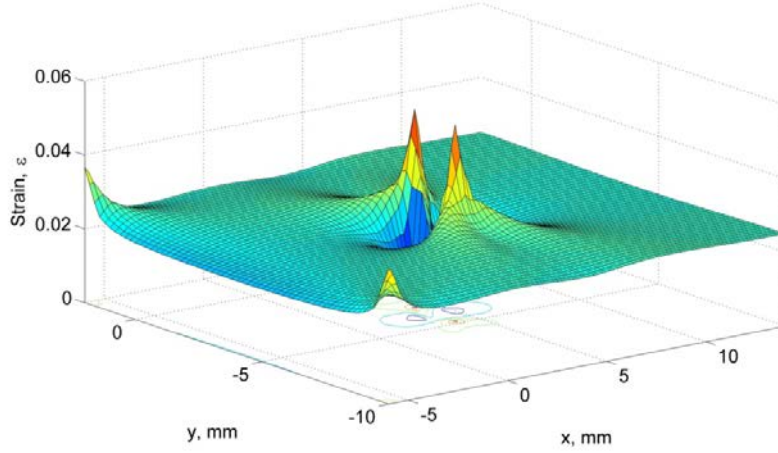
Fig. 7   A 3-D surface plot of the finite-element analysis (FEA) strain field. The strain
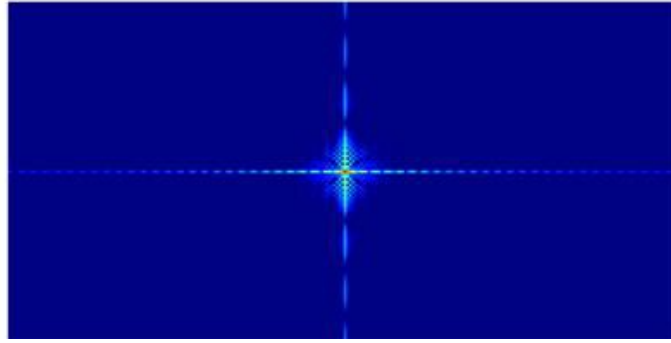spikes near the center hole

Fig. 8   The 2-D Fourier transform of the strain field

As stated, to measure such a sparse event, conventional SHM systems require several emplaced
or embedded sensors. Most of these sensors collect data that, in the new CS context, may not add
much to the accuracy or robustness of the damage detection process. In addition, the strain
measurement has to be acquired by a costly and lengthy measurement procedure, which seems to
be a waste of resources. Hence, one might ask whether there is a clever way of obtaining the
compressed version of the strain field directly by taking only a small number of measurements,
such as a few large Fourier coefficients, as shown in Fig. 8. The desire to acquire full-field
information from very few measurements is the major motivation of this research. In the
following section, we present a model problem that employs the compressed sensing strategy.

7. Problem Formulation

Consider a rectangular plate with a fastener hole at the center. The plate is subjected to a boundary condition that resulted in the strain field shown in Fig. 9. Figure 10 shows the FEA strain field in gray scale. Suppose we wish to monitor the state of strain using optical fiber Bragg grating (or FBG) sensors embedded in the plate. For the sake of simplicity, we assume that the FBGs are embedded in the radial direction, as shown by the yellow lines in Fig. 10. The yellow lines are the direction along which strain is being measured. We considered FBGs here; however, strain gages emplaced along these lines can also be envisioned. FBGs are strain-measuring sensors that use the principle of low coherence optical interferometry. We assume that L number of FBG sensors with m measuring nodes are embedded in a star-shaped radial arrangement through the center of the hole. Our goal is to recover the full-field strain (x, y) from discrete strain samples measured at $L \times m$ measuring nodes (Fig. 11). The $L \times m$ measuring nodes on the FBG are sensor arrays in which a number of distributed nodes acquire data and transmit to a back-end decoder or storage unit. All these sensors observe a related phenomenon (strain) and, as such, the ensemble of measurements they acquire is assumed to possess some joint structure or sparsity. In addition, from Fig. 7 we may conclude that the ensemble is sparse in some domain.
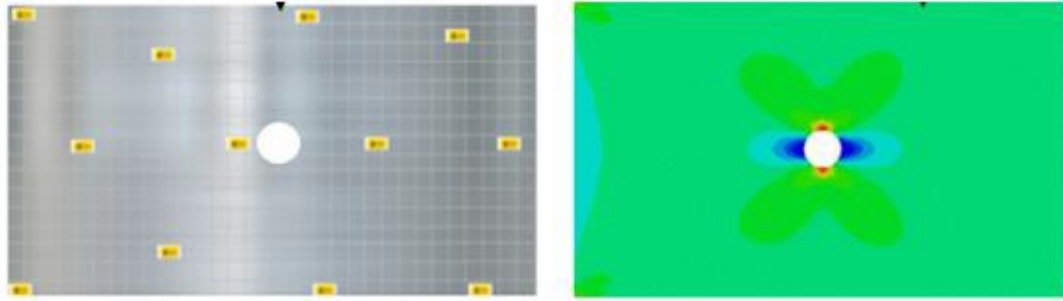

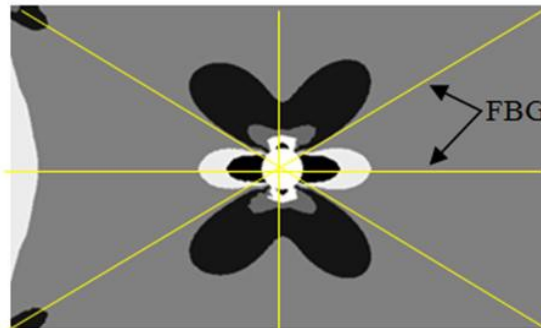Fig. 9   Plate with center hole and FEA simulation

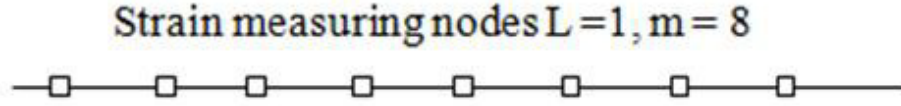Fig. 10   Sampling strain data along radial lines

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Fig. 11 Schematic of measuring nodes

To develop the joint sparsity model, we denote each sensor measurement in the ensemble by $\varepsilon_j \in R^N$, $j \in 1, 2, \ldots, J$ as sensor nodes. We assume that there exists a known sparse dictionary Ψ $\in R^N$ in which the strain ε_j can be sparsely represented. Denote by Φ the measurement matrix for signal j; φ_j is $M_j \times N$. For $L = 2, 5, 7, 9$ we used the l_1-norm solution to recover the original signals as shown in the Results section.

8. Results

Figure 12 shows a 194- $\times$ 194-pixel strain image recovered using l_1-norm. The main program code used to obtain the image is given in Appendix A. The l_1-norm is the convex relaxation of the analytically intractable l_0-norm. The reconstruction is obtained by maximizing the sparsity of the strain field in the Fourier domain. The 4 strain images shown in Fig. 12 are reconstructed from a limited number of Fourier samples taken along radial lines L as shown in Fig. 11. The number of samples used to reconstruct these strain images is much smaller than what would have been needed. The accuracy of reconstruction increases with the number of Fourier samples. Starting at $L = 2$, we see that the perturbation near the hole starts to appear clearly. A typical aerospace structure has a larger number of embedded FBGs than the numbers used in this example. With $L = 9$, one can almost clearly see the location of the peak strain (the spike). The results presented herein, in the current form, may not have the resolution and accuracy for use in a stand-alone SHM system; nonetheless, this approach might be implemented within passive system that serves as a warning mechanism to trigger a more robust system that performs an active scan of the area flagged by the sparse sensing unit. A step-by-step summary of tasks performed in this project are shown in Appendix B. Work is still in progress to further refine and extend this approach to piezoelectric signals, such as acoustic emission and ultrasonic guided wave SHM methods.
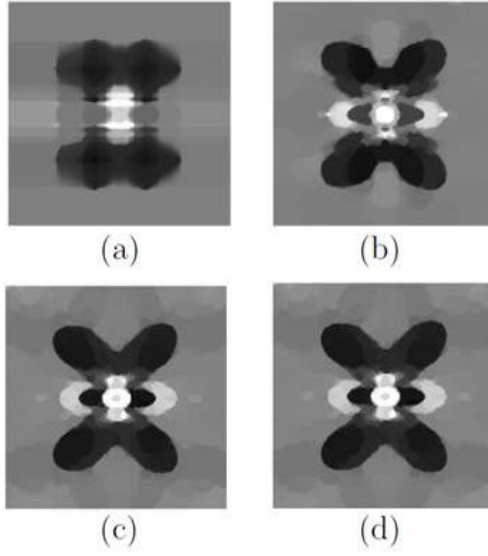
Fig. 12   Reconstructed strain images using (a)
L = 2, (b) L = 5, (c) L = 7, and (d) L = 9

9. References

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Appendix A. Basis Pursuit

The routine shown below solves the l_1 problem using the basis pursuit.

```
% Solve using CVX
cvx_begin
    variable xp(n)
    minimize (norm(xp, 1));
    subject to
        A * xp == y;
cvx_end

% CS example using CVX
% Random sampling matrix
% Representation basis is canonical
% Recovery using l1-magic

n = 512; % Signal length
s = 25;    % Sparsity
C = 5;     % Constant
m = C*s; % Number of measurements

f = get_sparse_fun(n, s);
A = get_A_random(n, m);

% Solve using l1-magic
path(path, './OPtimization');
x0 = pinv(A) * y; % initial guess
xp = l1eq_pd(x0, A, [], y, 1e-3);

norm(f - xp)/norm(f)
plot(f) hold on
plot(xp, 'r.')
legend('Original', 'Recovered')
```

Appendix B. Task Summary

This appendix summarizes the approach adopted while working on this project.

1. Research the structure of mechanical diagnostic signals, and determine if these signals possess sparsity and structure that can be exploited during signal acquisition. Explore the concept of distributed coding for a multisignal ensemble.

2. Research published dictionary of basis vectors in which mechanical metrics have sparse representation, and obtain a basis vector in which the joint measurement is sparse. Considered in this study are Fourier, discrete cosine, and wavelet basis vectors.

3. Develop recovery algorithms to reconstruct the original mechanical signal using convex optimization. Compare L1-minimization, matching pursuit, iterative thresholding, and total variation minimization.

4. Consider the model problem of reconstructing the 2-dimensional (2-D) strain field from incomplete measurement. Develop a finite element model of a rectangular plate with a center hole subjected to axial load and obtain nodal strain data.

5. Obtain the 2-D fast-Fourier transform of the strain image. Then acquire 1% of the nodal strain values (surface nodes) from the 2-D Fourier transform of the strain image. Then, using the L1 minimization, recover the full-field image and compare with the original finite element simulation image.

6. Explore concepts from continuum mechanics to develop a deterministic physics-based measuring matrix to subsample diagnostic data. Compare with random measurement matrices.

7. Explore sensing technologies (sensors) that directly acquire compressed samples from the field of measurement.

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